To Solve a Rational Inequality

1. Find any replacements for which the rational expression is undefined.
2. Change the inequality symbol to an equals sign and solve the related equation.
3. Use the numbers found in steps (1) and (2) to divide the number line into intervals.
4. Substitute a test value from each interval into the inequality. If the number is a solution, then the interval to which it belongs is part of the solution set. If solving graphically, examine the graph to determine the intervals that satisfy the inequality.
5. Select the interval(s) and any endpoints for which the inequality is satisfied and write set-builder notation or interval notation for the solution set. If the inequality symbol is $\leq$ or $\geq$, then the solutions from step (2) are also included in the solution set. Those numbers found in step (1) should be excluded from the solution set, even if they are solutions from step (2).

8.9 Exercise Set

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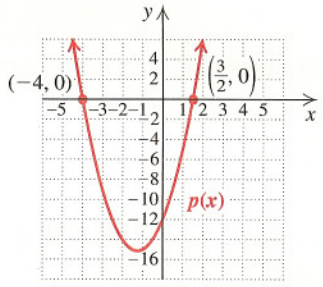
Concept Reinforcement *Classify each of the following statements as either true or false.*

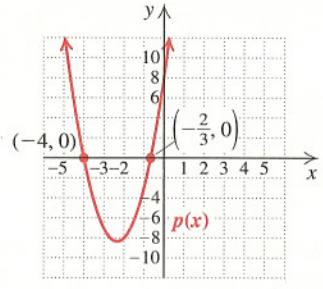
1. The solution of $(x - 3)(x + 2) \leq 0$ is $[-2, 3]$. True
2. The solution of $(x + 5)(x - 4) \geq 0$ is $[-5, 4]$. False
3. The solution of $(x - 1)(x - 6) > 0$ is $\{x \mid x < 1 \text{ or } x > 6\}$. True
4. The solution of $(x + 4)(x + 2) < 0$ is $(-4, -2)$. True
5. To solve $\frac{x + 2}{x - 3} < 0$ using intervals, we divide the number line into the intervals $(-\infty, -2)$ and $(-2, \infty)$. False
6. To solve $\frac{x - 5}{x + 4} \geq 0$ using intervals, we divide the number line into the intervals $(-\infty, -4)$, $(-4, 5)$, and $(5, \infty)$. True

Determine the solution set of each inequality from the given graph.

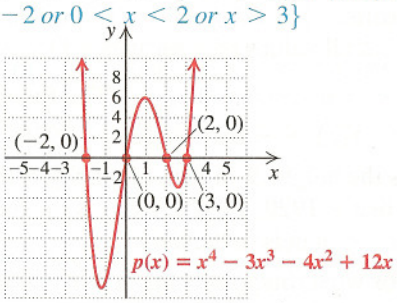
7. $p(x) \leq 0$ $[-4, \frac{3}{2}]$, or $\{x \mid -4 \leq x \leq \frac{3}{2}\}$



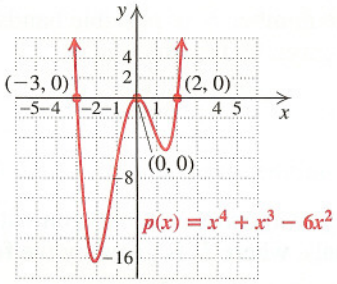
8. $p(x) < 0$ $(-4, -\frac{2}{3})$, or $\{x \mid -4 < x < -\frac{2}{3}\}$



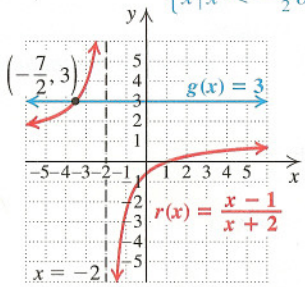
9. $x^4 + 12x > 3x^3 + 4x^2$ $(-\infty, -2) \cup (0, 2) \cup (3, \infty)$, or $\{x \mid x < -2 \text{ or } 0 < x < 2 \text{ or } x > 3\}$



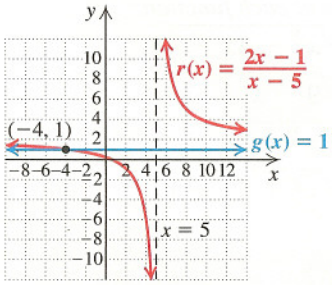
10. $x^4 + x^3 \geq 6x^2$ $(-\infty, -3] \cup \{0\} \cup [2, \infty)$, or $\{x \mid x \leq -3 \text{ or } x = 0 \text{ or } x \geq 2\}$



11. $\dfrac{x-1}{x+2} < 3$ $\left(-\infty, -\frac{7}{2}\right) \cup (-2, \infty)$, or $\left\{x \mid x < -\frac{7}{2} \text{ or } x > -2\right\}$



12. $\dfrac{2x-1}{x-5} \geq 1$ $(-\infty, -4] \cup (5, \infty)$, or $\{x \mid x \leq -4 \text{ or } x > 5\}$



Solve.

13. $(x + 4)(x - 3) < 0$ $(-4, 3)$, or $\{x \mid -4 < x < 3\}$

14. $(x + 5)(x + 2) > 0$ $(-\infty, -5) \cup (-2, \infty)$, or $\{x \mid x < -5 \text{ or } x > -2\}$

15. $(x + 7)(x - 2) \geq 0$ ⊡

16. $(x - 1)(x + 4) \leq 0$ $[-4, 1]$, or $\{x \mid -4 \leq x \leq 1\}$

17. $x^2 - x - 2 > 0$ ⊡

18. $x^2 + x - 2 < 0$ $(-2, 1)$, or $\{x \mid -2 < x < 1\}$

Aha! **19.** $x^2 + 4x + 4 < 0$ $\varnothing$

20. $x^2 + 6x + 9 < 0$ $\varnothing$

21. $x^2 - 4x < 10$ ⊡

22. $x^2 + 6x > -6$ ⊡

23. $3x(x + 2)(x - 2) < 0$ $(-\infty, -2) \cup (0, 2)$, or $\{x \mid x < -2 \text{ or } 0 < x < 2\}$

24. $5x(x + 1)(x - 1) > 0$ $(-1, 0) \cup (1, \infty)$, or $\{x \mid -1 < x < 0 \text{ or } x > 1\}$

25. $(x - 1)(x + 2)(x - 4) \geq 0$ $[-2, 1] \cup [4, \infty)$, or $\{x \mid -2 \leq x \leq 1 \text{ or } x \geq 4\}$

26. $(x + 3)(x + 2)(x - 1) < 0$ $(-\infty, -3) \cup (-2, 1)$, or $\{x \mid x < -3 \text{ or } -2 < x < 1\}$

27. $4.32x^2 - 3.54x - 5.34 \leq 0$ ⊡

28. $7.34x^2 - 16.55x - 3.89 \geq 0$ ⊡

29. $x^3 - 2x^2 - 5x + 6 < 0$ ⊡

30. $\frac{1}{3}x^3 - x + \frac{2}{3} > 0$ ⊡

31. For $f(x) = 7 - x^2$, find all x-values for which $f(x) \geq 3$. $[-2, 2]$, or $\{x \mid -2 \leq x \leq 2\}$

32. For $f(x) = 14 - x^2$, find all x-values for which $f(x) > 5$. $(-3, 3)$, or $\{x \mid -3 < x < 3\}$

33. For $g(x) = (x - 2)(x - 3)(x + 1)$, find all x-values for which $g(x) > 0$. ⊡

34. For $g(x) = (x + 3)(x - 2)(x + 1)$, find all x-values for which $g(x) < 0$. ⊡

35. For $F(x) = x^3 - 7x^2 + 10x$, find all x-values for which $F(x) \leq 0$. ⊡

36. For $G(x) = x^3 - 8x^2 + 12x$, find all x-values for which $G(x) \geq 0$. $[0, 2] \cup [6, \infty)$, or $\{x \mid 0 \leq x \leq 2 \text{ or } x \geq 6\}$

Solve.

37. $\dfrac{1}{x+5} < 0$ $(-\infty, -5)$, or $\{x \mid x < -5\}$

38. $\dfrac{1}{x+4} > 0$ $(-4, \infty)$, or $\{x \mid x > -4\}$

39. $\dfrac{x+1}{x-3} \geq 0$ ⊡

40. $\dfrac{x-2}{x+4} \leq 0$ $(-4, 2]$, or $\{x \mid -4 < x \leq 2\}$

41. $\dfrac{x+1}{x+6} \geq 1$ $(-\infty, -6)$, or $\{x \mid x < -6\}$

42. $\dfrac{x-1}{x-2} \leq 1$ $(-\infty, 2)$, or $\{x \mid x < 2\}$

43. $\dfrac{(x-2)(x+1)}{x-5} \leq 0$ ⊡

44. $\dfrac{(x+4)(x-1)}{x+3} \geq 0$ ⊡

45. $\dfrac{x}{x+3} \geq 0$ ⊡

46. $\dfrac{x-2}{x} \leq 0$ $(0, 2]$, or $\{x \mid 0 < x \leq 2\}$

47. $\dfrac{x-5}{x} < 1$ $(0, \infty)$, or $\{x \mid x > 0\}$

48. $\dfrac{x}{x-1} > 2$ $(1, 2)$, or $\{x \mid 1 < x < 2\}$

49. $\dfrac{x-1}{(x-3)(x+4)} \leq 0$ ⊡

50. $\dfrac{x+2}{(x-2)(x+7)} \geq 0$ ⊡

⊡ Answers to Exercises 15, 17, 21, 22, 27–30, 33–35, 39, 43–45, 49, and 50 are on pp. IA-20 and IA-21.

51. For $f(x) = \dfrac{5 - 2x}{4x + 3}$, find all x-values for which $f(x) \geq 0$. $\left(-\frac{3}{4}, \frac{5}{2}\right]$, or $\left\{x \mid -\frac{3}{4} < x \leq \frac{5}{2}\right\}$

52. For $g(x) = \dfrac{2 + 3x}{2x - 4}$, find all x-values for which $g(x) \geq 0$. $\left(-\infty, -\frac{2}{3}\right] \cup (2, \infty)$, or $\left\{x \mid x \leq -\frac{2}{3} \text{ or } x > 2\right\}$

53. For $G(x) = \dfrac{1}{x - 2}$, find all x-values for which $G(x) \leq 1$. $(-\infty, 2) \cup [3, \infty)$, or $\{x \mid x < 2 \text{ or } x \geq 3\}$

54. For $F(x) = \dfrac{1}{x - 3}$, find all x-values for which $F(x) \leq 2$. $(-\infty, 3) \cup \left[\frac{7}{2}, \infty\right)$, or $\left\{x \mid x < 3 \text{ or } x \geq \frac{7}{2}\right\}$

TW **55.** Explain how any quadratic inequality can be solved by examining a parabola.

TW **56.** Describe a method for creating a quadratic inequality for which there is no solution.

SKILL REVIEW

To prepare for Section 9.1, review function notation (Section 2.1).

Graph each function. [1.5], [2.1]

57. $f(x) = x^3 - 2$ ⊡ **58.** $g(x) = \dfrac{2}{x}$ ⊡

59. If $f(x) = x + 7$, find $f\left(\dfrac{1}{a^2}\right)$. [2.1] $\dfrac{1}{a^2} + 7$

60. If $g(x) = x^2 - 3$, find $g\left(\sqrt{a - 5}\right)$. [2.1], [7.1] $a - 8$

61. If $g(x) = x^2 + 2$, find $g(2a + 5)$. [2.1], [5.2] $4a^2 + 20a + 27$

62. If $f(x) = \sqrt[4]{4x + 1}$, find $f(3a - 5)$. [2.1] $\sqrt{12a - 19}$

SYNTHESIS

TW **63.** Step (5) on p. 672 states that even when the inequality symbol is $\leq$ or $\geq$, the solutions from step (2) are not always part of the solution set. Why?

TW **64.** Describe a method that could be used to create quadratic inequalities that have $(-\infty, a] \cup [b, \infty)$ as the solution set.

Find each solution set.

65. $x^4 + x^2 < 0$ $\varnothing$ **66.** $x^4 + 2x^2 \geq 0$ $(-\infty, \infty)$, or $\mathbb{R}$

67. $x^4 + 3x^2 \leq 0$ $\{0\}$ **68.** $\left|\dfrac{x + 2}{x - 1}\right| \leq 3$ ⊡

69. *Total Profit.* Derex, Inc., determines that its total-profit function is given by

$$P(x) = -3x^2 + 630x - 6000.$$

a) Find all values of x for which Derex makes a profit. $(10, 200)$, or $\{x \mid 10 < x < 200\}$

b) Find all values of x for which Derex loses money. $[0, 10) \cup (200, \infty)$, or $\{x \mid 0 \leq x < 10 \text{ or } x > 200\}$

70. *Height of a Thrown Object.* The function

$$S(t) = -16t^2 + 32t + 1920$$

gives the height S, in feet, of an object thrown from a cliff that is 1920 ft high. Here t is the time, in seconds, that the object is in the air. $\{t \mid 0 \text{ sec} < t < 2 \text{ sec}\}$

a) For what times does the height exceed 1920 ft?

b) For what times is the height less than 640 ft? $\{t \mid t > 10 \text{ sec}\}$

71. *Number of Handshakes.* There are n people in a room. The number N of possible handshakes by the people is given by the function

$$N(n) = \frac{n(n - 1)}{2}. \quad \{n \mid n \text{ is an integer and } 12 \leq n \leq 25\}$$

For what number of people n is $66 \leq N \leq 300$?

72. *Number of Diagonals.* A polygon with n sides has D diagonals, where D is given by the function

$$D(n) = \frac{n(n - 3)}{2}.$$

Find the number of sides n if $27 \leq D \leq 230$. $\{n \mid n \text{ is an integer and } 9 \leq n \leq 23\}$

Use a graphing calculator to graph each function and find solutions of $f(x) = 0$. Then solve the inequalities $f(x) < 0$ and $f(x) > 0$.

73. $f(x) = x + \dfrac{1}{x}$ ⊡

74. $f(x) = x - \sqrt{x}, x \geq 0$ ⊡

75. $f(x) = \dfrac{x^3 - x^2 - 2x}{x^2 + x - 6}$ ⊡

76. $f(x) = x^4 - 4x^3 - x^2 + 16x - 12$ ⊡

Find the domain of each function

77. $f(x) = \sqrt{x^2 - 4x - 45}$ ⊡

78. $f(x) = \sqrt{9 - x^2}$ ⊡

79. $f(x) = \sqrt{x^2 + 8x}$ ⊡

80. $f(x) = \sqrt{x^2 + 2x + 1}$ ⊡

Try Exercise Answers: Section 8.9
7. $\left[-4, \frac{3}{2}\right]$, or $\left\{x \mid -4 \leq x \leq \frac{3}{2}\right\}$
21. $\left(2 - \sqrt{14}, 2 + \sqrt{14}\right)$, or $\left\{x \mid 2 - \sqrt{14} < x < 2 + \sqrt{14}\right\}$
31. $[-2, 2]$, or $\{x \mid -2 \leq x \leq 2\}$
35. $(-\infty, 0] \cup [2, 5]$, or $\{x \mid x \leq 0 \text{ or } 2 \leq x \leq 5\}$
41. $(-\infty, -6)$, or $\{x \mid x < -6\}$

⊡ Answers to Exercises 57, 58, 68, and 73–80 are on p. IA-21.